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AN OPTIMAL STAFFING AND SCHEDULING APPROACH IN OPEN SHOP ENVIRONMENT

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Abstract

This paper is concerned with the defining of staff number and corresponding work schedule in open shop environment. The aim is to determine the optimal or Pareto-optimal number of staff for given open shop job that is performed on a given number of machines and operations. Three optimization models are proposed with different objective functions to illustrate the flexibility of the proposed approach. Its applicability is demonstrated by numerical testing for real life problem.

Key words: integer programming, open shop, staffing, work schedule

1. Introduction. Many management decisions involve trying to make the most effective use of an organization's resources. Linear programming is a widely used mathematical technique designed to help managers to plan and take the decisions about optimal allocations of resources. Planning the personnel tends to be non-trivial due to the many sources of variability inherent in real-life service systems [1]. Optimization of staff is not about reducing the staff number but about finding the right number of staff at the right time in the right place. Staff scheduling is a common problem to most organizations, either from the service sector or industry. The staffing concerns problems of maximizing the utilization of manpower to perform a set of activities in order to achieve a certain goal [2]. Although the staff scheduling problem has been intensively explored in the literature, studies usually focus on solving very particular problems that derive from

Input data

	Operation 1 on Machine 1	Operation 2 on Machine 2	. . .	Operation m on Machine m
Task processing time	t_1	t_2	. . .	t_i
Number of operators needed to service of machine	w_1	w_2	. . .	w_i

practical needs [3–6]. Most of the proposed models deal with heuristic approaches to optimize staffing and scheduling problems [7,8].

In contrast to known heuristic approaches, the current paper proposes integrated approach for optimal or Pareto-optimal staffing determination. This approach aims to define the minimum staff number and scheduling corresponding to minimum staff idle to perform a given job in open shop environment. The proposed approach is numerically tested for real staff problem to demonstrate its applicability.

2. Problem settings. Investigated open shop staffing problem addresses a single job consisting of given number of operations that have to be processed on a number of machines. Each operation is assigned to a particular machine with known processing time. Each machine is served by a given number of operators. The job operations can be processed in any order in sequence or in parallel. The processing times are independent of the processing sequence. There is only one of each type of machine and the machines operators have competence to serve all of the machines. The corresponding to this problem input data (operations, machines, processing times and needed operators) are shown in Table 1.

Different types of management goals can be considered – minimizing the overall staff idle, minimizing of the total processing time or simultaneous determination of the minimum processing time and minimum staff number.

3. Optimization model. The input data of the described problem can be represented in x, y coordinates, where the x coordinate axis illustrates the processing time duration (T, h) while the y coordinate axis is used to illustrate the total number of operators (W) serving the machines (Fig. 1).

Each operation is illustrated by a rectangle with length represented on the x coordinate axis (processing time) and width represented on the y coordinate axis (number of operators). An obligatory requirement for correct modelling of the described problem is that all rectangles representing operations should not overlap. This is achieved by the following constraints:

(1) $$x_j \geq 0, \ integer, \ j \in J$$

(2) $$y_j \geq 0, \ integer, \ j \in J$$

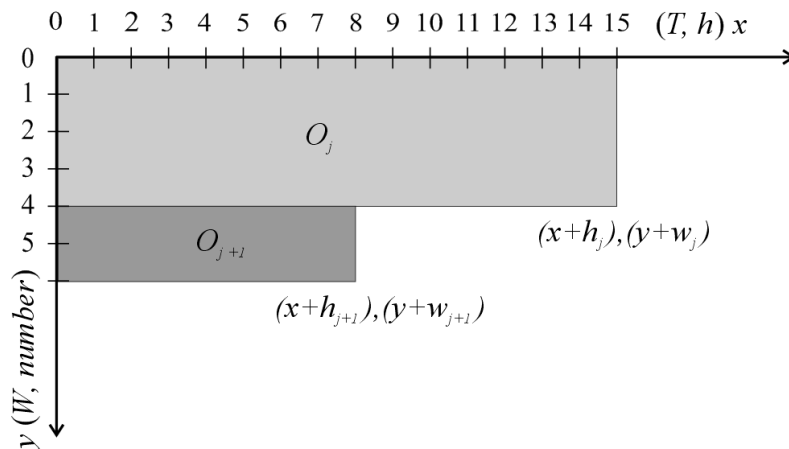(3) $$x_j + h_j \leq T, \quad j \in J$$

Fig. 1. An illustration of the used approach for the described problem

$$(4) \qquad y_j + w_j \leq W, \quad j \in J$$

$$(5) \qquad \max\{h_j\} \leq T \leq \sum_{j=1}^{J} h_j$$

$$(6) \qquad \max\{w_j\} \leq W \leq \sum_{j=1}^{J} w_j$$

$$(7) \qquad x_j + h_j \leq x_m + (1 - l_{jm})T_{\max}, \quad j, m \in J$$

$$(8) \qquad y_j + w_j \leq y_m + (1 - b_{jm})W_{\max}, \quad j, m \in J$$

$$(9) \qquad l_{jm} + l_{mj} + b_{jm} + b_{mj} = 1, \quad j \in J,$$

where $J = \{1, 2, \ldots, n\}$ – set of operations; w_j and h_j – needed operators and duration of jth operation; x_j and y_j – coordinates of the upper left corner of the rectangle representing the jth operation; l_{jm} is equal to 1, if jth operation is located on left to the mth operation and 0 otherwise [9]; b_{jm} is equal to 1, if jth operation is below the mth operation and 0 otherwise [9].

The overall staff idle for completion of all jobs is determined by the statement:

$$(10) \qquad I = (W.T) - \sum_{j=1}^{J} w_j h_j, \quad j \in J$$

where: I – staff idle, W – total number of operators serving the machines, $W_{\min} \leq W \leq W_{\max}$, $W_{\min} = \max\{w_j\}$ – minimal number of operators serving the machines (for sequential processing scenario), $W_{\max} = \sum w_j$ – maximal number of operators to serve machines for parallel processing scenario, T – overall job processing time, w_j – number of operators serving jth machine, h_j – processing time on jth machine.

The model formulated above can be used for determination of staff number and corresponding work schedules conforming to different management requirements by formulation of optimization tasks with proper objective functions. Typical problems for optimal job staffing and scheduling are determination of minimum staff idle [10], determination of minimum processing time or simultaneous determination of minimum staff idle and minimum staff number. The corresponding optimization tasks for using these problems are:

Task 1: Determination of operators' number to achieve minimum staff idle:

$$(11) \qquad\qquad \min\{I_i\}$$

subject to restrictions (1)–(10).

Task 2: Determination of operators' number providing minimum processing time:

$$(12) \qquad\qquad \min\{T_i\}$$

subject to restrictions (1)–(10).

Task 3: Simultaneous determination of minimum staff idle and minimum staff number:

$$(13) \qquad\qquad \begin{aligned} \min\{T_i\} \\ \min\{W_i\} \end{aligned}$$

subject to the same restrictions (1)–(10).

These optimizations tasks demonstrate the flexibility of the proposed model to adjust to different real life open shop scheduling problems. Using of multicriteria optimization leads to more informed and better decisions when conflicting objectives exist. In multicriteria optimization methods the decision maker plays a major role in providing information for his preferences that influence the final solution [11–14]. The most widely used methods are based on a priori expressing of the decision maker's preferences [15].

4. Numerical testing. The proposed modelling approach for optimal staffing in open shop environment is numerically tested for a real manufacturing problem with input data shown in Table 2.

Task 1 and Task 2 are solved sequentially for each value of i. The index i takes values from 4 (for sequential processing scenario) to 10 (for parallel processing scenario). The formulated single-objective Task 1 and Task 2 are solved by means of branch and bound algorithm implemented in Lingo solver [16]. The multicriteria Task 3 is solved by lexicographic solution method [15]. The solution results of formulated Task 1, Task 2 and Task 3 are summarized in Table 3.

5. Results, analysis and discussion. The results of solutions for Task 1 show that the minimal staff idle is equal to 16 h which corresponds to 6 operators. The solutions for Task 2 show that the minimal processing time of 20 h is

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	Operation 1 on Machine 1	Operation 2 on Machine 2	Operation 3 on Machine 3	Operation 4 on Machine 4
Task processing time	10	15	20	4
Number of operators serving the machine	3	4	2	1
Number of machines	1	1	1	1

Table 3

Solution results for Task 1, Task 2 and Task 3

	Operators W, number	Staff idle I, man–h	Processing time T, h
Task 1	6	16	25
Task 2	9	46	20
	10	66	20
Task 3	5	35	41

reached for 9 operators and for 10 operators. Obviously, the better alternative in this case is using 9 operators. The multicriteria Task 3 is solved by the lexicographic method. This method is based on a priori aggregation of preference information by arranging of objective functions in order of importance. Then a sequence of single objective optimization problems are solved one at a time by order of their importance. For Task 3 the used lexicographic ordering is defining of processing time as more important than defining staff number. At the first step of lexicographic method a single objective optimization task for minimization of overall processing time is solved and the solution defines minimum processing time of 20 h. This value is used to define an upper limit for the processing time that is used at the second stage of lexicographic method. The lexicographic method implementation of Task 3 uses value of 40 h. The final Pareto-optimal solution determines the overall processing time equal to 35 h with staff number equal to 5 corresponding to 41 h staff idle.

The advantage of proposed approach is the ability to use different formulation of optimization tasks based on different objective functions depending on the company strategy. This modelling approach and the formulated optimization tasks are not heuristic in the sense that they maintain a provable upper and lower bound on the (globally) optimal objective value.

6. Conclusion. The paper describes an approach for staffing and scheduling in open shop environment. The proposed model can be used with different objectives (single or multicriteria) depending on the management decision making strategy. A real life industry staffing problem is used to demonstrate the practical applicability of the proposed integrated approach. The described optimization

model can be easily adjusted to address different real life staff scheduling problems from different application areas. As future developments of the proposed approach could be related with using of other optimization criteria and different multicriteria solution methods based on a posteriori articulation of preference information (as e-constraint or weighted sum methods) to get Pareto-optimal solutions.

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